First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

MAT264 - Non-parametric Statistics
(Statistik Tak Berparameter)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TWENTY TWO (22) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi DUA PULUH DUA (22) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **SIX (6)** questions.

*[**Arahan** : Jawab **ENAM (6)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

A random sample of 12 persons are interviewed to estimate median annual gross income in a certain economically depressed town. The data are as follow:

9800	10200	9300	8700	15200	6900
8600	9600	12200	15500	11600	7200

(a) Can you conclude that the data are collected randomly?

(b) (i) Using the Wilcoxon signed-rank test at the 5% significance level, can you conclude that the median annual gross income of the town is 11000?

(ii) Perform the test of part (a) using the sign test at the 5% significance level.

(iii) Compare your conclusions from parts (i) and (ii).

(iv) Construct a 95% confidence interval for median by using the Wilcoxon signed-rank test.

(c) Use the most appropriate test to test that the distribution can be assumed to be normal with mean 10000 and standard deviation 2000.

[30 marks]

Soalan 1

Satu sampel rawak seramai 12 orang ditemuduga untuk menganggar median pendapatan kasar tahunan di sebuah bandar tertentu yang mengalami kemerosotan ekonomi. Data adalah seperti berikut:

9800	*10200*	*9300*	*8700*	*15200*	*6900*
8600	*9600*	*12200*	*15500*	*11600*	*7200*

(a) Bolehkah anda menyimpulkan bahawa data adalah dikumpul secara rawak?

(b) (i) Gunakan Wilcoxon pangkat-bertanda, pada aras keertian 5%, bolehkah anda menyimpulkan bahawa median pendapatan kasar tahunan di bandar tersebut adalah 11000?

(ii) *Lakukan ujian di bahagian (a) dengan menggunakan ujian tanda pada aras keertian 5%.*

(iii) *Bandingkan kesimpulan anda di bahagian (i) dan (ii).*

(iv) *Binakan suatu selang keyakinan 95% untuk median dengan menggunakan ujian Wilcoxon pangkat-bertanda.*

(c) *Gunakan ujian yang paling sesuai untuk menguji bahawa taburan boleh dianggap normal dengan min 10000 dan sisihan piawai 2000.*

[30 markah]

Question 2

A psychological study involved the rating of rats along a dominance submissiveness continuum. In order to determine the reliability of the ratings, the ranks given by two different observers were tabulated below.

Animal	Rank observer A	Rank observer B
A	12	15
B	2	1
C	3	7
D	1	4
E	4	2
F	5	3
G	14	11
H	11	10
I	6	5
J	9	9
K	7	6
L	10	12
M	15	13
N	8	8
O	13	14
P	16	16

(a) Are the ratings agreeable? Explain your answer.

(b) Determine whether there is a significant relationship between the ranks given by the two different observers.

[20 marks]

Soalan 2

Suatu kajian psikologi melibatkan penarafan tikus sepanjang kontinum penyerahan dominasi. Untuk menentukan kebolehpercayaan penarafan, pangkat yang diberikan oleh dua pemerhati yang berbeza telah ditabulasi di bawah.

Haiwan	***Pangkat pemerhati A***	***Pangkat pemerhati B***
A	*12*	*15*
B	*2*	*1*
C	*3*	*7*
D	*1*	*4*
E	*4*	*2*
F	*5*	*3*
G	*14*	*11*
H	*11*	*10*
I	*6*	*5*
J	*9*	*9*
K	*7*	*6*
L	*10*	*12*
M	*15*	*13*
N	*8*	*8*
O	*13*	*14*
P	*16*	*16*

(a) Adakah penilaian itu sesuai? Terangkan jawapan anda.

(b) Tentukan sama ada terdapat hubungan yang signifikan antara pangkat yang diberikan oleh dua pemerhati yang berbeza.

[20 markah]

Question 3

(a) Explain what determines whether to use Wilcoxon signed-rank test or the Wilcoxon rank sum test.

(b) Diet A was given to a group of 10 overweight boys between the ages of 8 and 10. Diet B was given to another independent group of 8 similar overweight boys. The weight loss is given below.

Diet A	7	2	3	-1	4	6	0	1	4
Diet B	5	6	4	7	8	9	7	2	

(i) Carry out Mann-Whitney test to test that the hypothesis that the diets are comparable effectiveness.

(ii) Perform the test of part (i) using normal approximation.

(iii) Comment the results obtained from parts (i) and (ii).

[15 marks]

Soalan 3

(a) Terangkan apa yang menentukan sama ada hendak menggunakan ujian Wilcoxon pangkat-bertanda atau ujian Wilcoxon jumlah pangkat.

(b) Diet A diberikan kepada sekumpulan 10 kanak-kanak lelaki berlebihan berat badan antara umur 8 dan 10. Diet B diberikan kepada kumpulan bebas yang lain sebanyak 8 lelaki berlebihan berat badan yang sama. Kehilangan berat badan diberikan di bawah.

Diet A	*7*	*2*	*3*	*-1*	*4*	*6*	*0*	*1*	*4*
Diet B	*5*	*6*	*4*	*7*	*8*	*9*	*7*	*2*	

(i) Jalankan ujian Mann-Whitney untuk menguji bahawa hipotesis bahawa kedua-dua diet mempunyai keberkesanan yang setanding.

(ii) *Lakukan ujian di bahagian (i) dengan menggunakan anggaran normal.*

(iii) *Komen hasil yang diperoleh daripada bahagian (i) dan (ii).*

[15 markah]

Question 4

The venerable auction house of Snootly & Snobs will soon be putting three fine 17^{th} and 18^{th} century violins, A, B and C, up for bidding. A certain musical arts foundation, wishing to determine which of these instruments to add to its collection, arranges to have them played by each of 10 concert violinists. The players are blindfolded, so that they can not tell which violin is which: and each plays the violins in a randomly determined sequence (BCA, ABC, etc). The violinists are not informed that the instruments are classic masterworks; all they know is that they are playing three different violins. After each violin is played, the player rates the instrument on a 10-point scale of overall excellence (1 = lowest, 10 = highest). The players are told that they can also give fractional ratings, such as 6.2 or 4.5, if they wish. The results are shown as below.

	Violinist									
Violin	1	2	3	4	5	6	7	8	9	10
A	9	9.5	5	7.5	9.5	7.5	8	7	8.5	6
B	7	6.5	7	7.5	5	8	6	6.5	7	7
C	6	8	4	6	7	6.5	6	4	6.5	3

(a) Perform a suitable analysis of the data.

(b) From the results obtained from part (a), what suggestion would you give to the musical arts foundation?

[10 marks]

Soalan 4

Rumah lelong yang dihormati Snootly & Snobs tidak lama lagi akan menempatkan tiga violin abad ke-17 dan ke-18, A, B dan C, untuk dibida. Sebuah yayasan seni muzik tertentu, yang ingin menambahkan salah satu instrumen ke dalam koleksinya, mengatur agar ketiga-tiga instrumen dimainkan oleh masing-masing 10 pemain biola konsert. Para pemain ditutup mata, supaya mereka tidak dapat membezakan antara biola-biola itu: dan masing-masing memainkan biola dalam urutan secara rawak (BCA, ABC, dll). Pemain vbiola tidak dimaklumkan bahawa instrumen itu adalah karya master klasik; semua yang mereka tahu ialah mereka bermain 3 biola yang berbeza. Selepas setiap biola dimainkan, pemain menilai instrumen pada skala 10 mata keseluruhan kecemerlangan (1 = terendah, 10 = tertinggi). Pemain diberitahu bahawa mereka juga boleh memberikan penarafan fraksional, seperti 6.2 atau 4.5, jika mereka mahu. Hasilnya ditunjukkan seperti di bawah.

	Pemain Biola									
Biola	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
A	*9*	*9.5*	*5*	*7.5*	*9.5*	*7.5*	*8*	*7*	*8.5*	*6*
B	*7*	*6.5*	*7*	*7.5*	*5*	*8*	*6*	*6.5*	*7*	*7*
C	*6*	*8*	*4*	*6*	*7*	*6.5*	*6*	*4*	*6.5*	*3*

(a) *Lakukan analisa yang sesuai kepada data tersebut.*

(b) *Dari hasil yang diperoleh daripada bahagian (a), apakah cadangan yang anda akan berikan kepada yayasan seni muzik?*

[10 markah]

Question 5

(a) Given that the test statistic of Friedman test is

$$F_r = \frac{12b}{k(k+1)}\sum\left(\overline{R}_j - \overline{R}\right)^2$$

show that the alternative test of Friedman test is

$$F_r = \frac{12}{bk(k+1)}\sum R_j^2 - 3b(k+1),$$

where

$b =$ Number of blocks, $k =$ Number of treatments,

$\overline{R}_j =$ Mean rank sum for the *j*th treatment, $\overline{R} =$ Mean of all rank, and

$R_j =$ Rank sum of the *j*th treatment, where the rank of each measurement is computed relative to its position within its own block.

(b) An experiment was run to determine whether four specific firing temperatures affect the density of a certain type of brick. The experiment led to the following data. Does the firing temperature affect the density of the bricks?

Temperature	100	125	150	175
Density	21.8	21.7	21.9	21.9
	21.9	21.4	21.8	21.7
	21.7	21.5	21.8	21.8
	21.7	21.4	21.8	21.4
	21.6		21.6	
	21.7		21.5	

[15 marks]

Soalan 5

(a) Diberi bahawa statistik ujian bagi Friedman ialah

$$F_r = \frac{12b}{k(k+1)}\sum\left(\overline{R}_j - \overline{R}\right)^2$$

tunjukkan bahawa ujian alternatif ujian Friedman ialah

$$F_r = \frac{12}{bk(k+1)}\sum R_j^2 - 3b(k+1),$$

di mana

$b =$ *bilangan blocks,* $k =$ *bilangan rawatan,*

$\overline{R}_j =$ *min jumlah pangkat bagi rawatan ke-j,* $\overline{R} =$ *min semua pangkat, dan*

$R_j =$ *Jumlah pangkat rawatan ke-j, di mana setiap ukuran pangkat dikira dengan perbandingan kedudukannya dalam blok sendiri.*

(b) *Eksperimen dijalankan untuk menentukan sama ada empat suhu pembakaran tertentu mempengaruhi ketumpatan jenis bata tertentu. Percubaan membawa kepada data berikut. Adakah suhu pembakaran mempengaruhi ketumpatan batu bata?*

Suhu	*100*	*125*	*150*	*175*
Ketumpatan	*21.8*	*21.7*	*21.9*	*21.9*
	21.9	*21.4*	*21.8*	*21.7*
	21.7	*21.5*	*21.8*	*21.8*
	21.7	*21.4*	*21.8*	*21.4*
	21.6		*21.6*	
	21.7		*21.5*	

[15 markah]

Question 6

A study was designed to test whether or not aggression is a function of anonymity. The study was conducted as a field experiment on Halloween. 300 children were observed unobtrusively as they made their rounds. Of these 300 children, 173 wore masks that completely covered their face, while 127 wore no masks. It was found that 101 children in the masked group displayed aggressive or antisocial behavior versus 36 children in the unmasked group. What conclusion can be drawn? By using $\alpha = 0.01$, state your conclusion in the terminology of the problem.

[10 marks]

Soalan 6

Satu kajian telah direka untuk menguji sama ada agresif adalah fungsi "anonymity". Kajian ini dijalankan sebagai ujikaji lapangan pada Halloween. 300 kanak-kanak diperhatikan ketika mereka membuat pusingan. Daripada 300 kanak-kanak ini, 173 memakai topeng yang menutupi wajah mereka, sementara 127 tidak memakai topeng. Adalah didapati bahawa 101 kanak-kanak dalam kumpulan bertopeng menunjukkan tingkah laku agresif atau antisosial berbanding 36 kanak-kanak dalam kumpulan yang tidak bertopeng. Apakah kesimpulan yang boleh dibuat? Dengan menggunakan $\alpha = 0.01$, nyatakan kesimpulan anda dalam terminologi masalah.

[10 markah]

APPENDIX

1. Sign Test:

Small sample: X = Number of (+) signs [or (–) signs]

Large sample: $z = \dfrac{(k+0.5) - n/2}{\sqrt{n}/2}$

2. Wilcoxon Signed-rank:

Small sample: $W = \min\left(\sum(+), \sum(-)\right)$

Large sample: $Z = \dfrac{W - \mu_W}{\sigma_W}$, $\mu_W = \dfrac{n(n+1)}{4}$, $\sigma_W = \sqrt{\dfrac{n(n+1)(2n+1)}{24}}$

3. Mann-Whitney Test:

Small sample: $U_1 = n_1 n_2 + \dfrac{n_2(n_1+1)}{2} - R_2$

$$U_2 = n_1 n_2 + \frac{n_1(n_2+1)}{2} - R_1$$

Large sample: $z = \dfrac{U - \frac{n_1 n_2}{2}}{\sqrt{\dfrac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$

4. The Median Test:

$$T = \frac{A/n_1 - B/n_2}{\sqrt{\hat{p}(1-)\hat{p}\left(1/n_1 + 1/n_2\right)}}$$

5. Chi-square Test:

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

6. Fisher's Exact Test:

$$P = \frac{(n_{11}+n_{12})!(n_{11}+n_{21})!(n_{21}+n_{22})!(n_{12}+n_{22})!}{n_{11}!n_{12}!n_{21}!n_{22}!n!}$$

7. McNemar's Test:

$$z = \frac{n_{12}-n_{21}}{\sqrt{n_{12}+n_{21}}}$$

8. Run Test:

Large sample: $$z = \frac{r - \left\{\left[(2n_1n_2)/(n_1+n_2)\right]+1\right\}}{\sqrt{\dfrac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}}$$

9. Wald-Walfowitz Runs Test:

Large Sample: $$z = \frac{r - \left(\dfrac{2n_1n_2}{n_1+n_2}+1\right)}{\sqrt{\dfrac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}}$$

10. Cox-Stuart Test:

X = Number of (+) signs [or (–) signs]

11. Kruskal-Wallis Test:

$$H = \frac{12}{n(n+1)}\sum\frac{R_j^2}{n_j} - 3(n+1)$$

12. Friedman F_r -Test:

$$F_r = \frac{12}{bk(k+1)}\sum R_j^2 - 3b(k+1)$$

13. Spearman's Rank Correlation Coefficient:

$$r_s = 1 - \frac{6\sum d^2}{n\left(n^2 - 1\right)}$$

14. Kendall's Tau Test

$$\hat{\tau} = \frac{S}{n\left(n-1\right)/2}$$

15. Kolmogorov-Smirnov One-Sample Test:

$$D = \sup_x | S\left(x\right) - F_o\left(x\right) |$$

16. Kolmogorov-Smirnov Two-Sample Test:

$$D = \max \left| S_1\left(x\right) - S_2\left(x\right) \right|$$

LIST OF TABLES

TABLE A-7 Critical Values for the Sign Test

	α			
n	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)
1	*	*	*	*
2	*	*	*	*
3	*	*	*	*
4	*	*	*	*
5	*	*	*	0
6	*	*	0	0
7	*	0	0	0
8	0	0	0	1
9	0	0	1	1
10	0	0	1	1
11	0	1	1	2
12	1	1	2	2
13	1	1	2	3
14	1	2	2	3
15	2	2	3	3
16	2	2	3	4
17	2	3	4	4
18	3	3	4	5
19	3	4	4	5
20	3	4	5	5
21	4	4	5	6
22	4	5	5	6
23	4	5	6	7
24	5	5	6	7
25	5	6	7	7

NOTES:

1. * indicates that it is not possible to get a value in the critical region.
2. Reject the null hypothesis if the number of the less frequent sign (x) is less than or equal to the value in the table.
3. For values of n greater than 25, a normal approximation is used with

$$z = \frac{(x + 0.5) - \left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}}$$

Table A.3 *d*-factors for Wilcoxon signed-rank test and confidence intervals for the median (α' = one-sided significance level, α'' = two-sided significance level)

n	d	Confidence coefficient	α''	α'
3	1	.750	.250	.125
4	1	.875	.125	.063
5	1	.938	.062	.031
	2	.875	.125	.063
6	1	.969	.031	.016
	2	.937	.063	.031
	3	.906	.094	.047
	4	.844	.156	.078
7	1	.984	.016	.008
	2	.969	.031	.016
	4	.922	.078	.039
	5	.891	.109	.055
8	1	.992	.008	.004
	2	.984	.016	.008
	4	.961	.039	.020
	5	.945	.055	.027
	6	.922	.078	.039
	7	.891	.109	.055
9	2	.992	.008	.004
	3	.988	.012	.006
	6	.961	.039	.020
	7	.945	.055	.027
	9	.902	.098	.049
	10	.871	.129	.065
10	4	.990	.010	.005
	5	.986	.014	.007
	9	.951	.049	.024
	10	.936	.064	.032
	11	.916	.084	.042
	12	.895	.105	.053
11	6	.990	.010	.005
	7	.986	.014	.007
	11	.958	.042	.021
	12	.946	.054	.027
	14	.917	.083	.042
	15	.898	.102	.051
12	8	.991	.009	.005
	9	.988	.012	.006
	14	.958	.042	.021
	15	.948	.052	.026
	18	.908	.092	.046
	19	.890	.110	.055
13	10	.992	.008	.004
	11	.990	.010	.005
	18	.952	.048	.024
	19	.943	.057	.029
	22	.906	.094	.047
	23	.890	.110	.055
14	13	.991	.009	.004
	14	.989	.011	.005
	22	.951	.049	.025
	23	.942	.058	.029
	26	.909	.091	.045
	27	.896	.104	.052
15	16	.992	.008	.004
	17	.990	.010	.005
	26	.952	.048	.024
	27	.945	.055	.028
	31	.905	.095	.047
	32	.893	.107	.054
16	20	.991	.009	.005
	21	.989	.011	.006
	30	.956	.044	.022
	31	.949	.051	.025
	36	.907	.093	.047
	37	.895	.105	.052
17	24	.991	.009	.005
	25	.989	.011	.006
	35	.955	.045	.022
	36	.949	.051	.025
	42	.902	.098	.049
	43	.891	.109	.054
18	28	.991	.009	.005
	29	.990	.010	.005
	41	.952	.048	.024
	42	.946	.054	.027
	48	.901	.099	.049
	49	.892	.108	.054
19	33	.991	.009	.005
	34	.989	.011	.005
	47	.951	.049	.025
	48	.945	.055	.027
	54	.904	.096	.048
	55	.896	.104	.052
20	38	.991	.009	.005
	39	.989	.011	.005
	53	.952	.048	.024
	54	.947	.053	.027
	61	.903	.097	.049
	62	.895	.105	.053
21	43	.991	.009	.005
	44	.990	.010	.005
	59	.954	.046	.023
	60	.950	.050	.025
	68	.904	.096	.048
	69	.897	.103	.052
22	49	.991	.009	.005
	50	.990	.010	.005
	66	.954	.046	.023
	67	.950	.050	.025
	76	.902	.098	.049
	77	.895	.105	.053
23	55	.991	.009	.005
	56	.990	.010	.005
	74	.952	.048	.024
	75	.948	.052	.026
	84	.902	.098	.049
	85	.895	.105	.052
24	62	.990	.010	.005
	63	.989	.011	.005
	82	.951	.049	.025
	83	.947	.053	.026
	92	.905	.095	.048
	93	.899	.101	.051
25	69	.990	.010	.005
	70	.989	.011	.005
	90	.952	.048	.024
	91	.948	.052	.026
	101	.904	.096	.048
	102	.899	.101	.051

Source: F. Wilcoxon, S. Katti, and R. A. Wilcox, *Critical Values and Probability Levels for the Wilcoxon Rank Sum Test and the Wilcoxon Signed Rank Test*, Pearl River, N.Y.: American Cyanamid Co., 1949; used by permission of American Cyanamid Company

Note: For $n > 25$ use $d \approx \frac{1}{2}[\frac{1}{2}n(n+1)+1-z\sqrt{n(n+1)(2n+1)/6}]$, where z is read from Table A.2.

TABLE A-10 Critical Values for Number of Runs G

Value of n_1	Value of n_2: 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
3	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
	6	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
4	1	1	1	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
	6	8	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10	10	10
5	1	1	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
	6	8	9	10	10	11	11	12	12	12	12	12	12	12	12	12	12	12	12
6	1	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
	6	8	9	10	11	12	12	13	13	13	13	14	14	14	14	14	14	14	14
7	1	2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
	6	8	10	11	12	13	13	14	14	14	14	15	15	15	16	16	16	16	16
8	1	2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
	6	8	10	11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
	6	8	10	12	13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10	1	2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11	1	2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
	6	8	10	12	13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
	6	8	10	12	13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
	6	8	10	12	14	15	16	17	18	19	19	20	20	21	21	22	22	23	23
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
	6	8	10	12	14	15	16	17	18	19	20	20	21	22	22	23	23	23	24
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
	6	8	10	12	14	15	16	18	18	19	20	21	22	22	23	23	24	24	25
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
	6	8	10	12	14	16	17	18	19	20	21	21	22	23	23	24	25	25	25
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
	6	8	10	12	14	16	17	18	19	20	21	22	23	23	24	25	25	26	26
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
	6	8	10	12	14	16	17	18	19	20	21	22	23	24	25	25	26	26	27
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
	6	8	10	12	14	16	17	18	20	21	22	23	23	24	25	26	26	27	27
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14
	6	8	10	12	14	16	17	18	20	21	22	23	24	25	25	26	27	27	28

NOTE:

1. The entries in this table are the critical G values, assuming a two-tailed test with a significance level of $\alpha = 0.05$.
2. The null hypothesis of randomness is rejected if the total number of runs G is less than or equal to the smaller entry or greater than or equal to the larger entry.

From "Tables for testing randomness of groupings in a sequence of alternatives," *The Annals of Mathematical Statistics,* Vol. 14, No. 1. Reprinted with permission of the Institute of Mathematical Statistics.

Appendix 3

Kolmogorov–Smirnov Tables

Critical values, $d_{alpha};(n)^a$, of the maximum absolute difference between sample $F_n(x)$ and population $F(x)$ cumulative distribution.

Number of trials, n	Level of significance, α			
	0.10	0.05	0.02	0.01
1	0.95000	0.97500	0.99000	0.99500
2	0.77639	0.84189	0.90000	0.92929
3	0.63604	0.70760	0.78456	0.82900
4	0.56522	0.62394	0.68887	0.73424
5	0.50945	0.56328	0.62718	0.66853
6	0.46799	0.51926	0.57741	0.61661
7	0.43607	0.48342	0.53844	0.57581
8	0.40962	0.45427	0.50654	0.54179
9	0.38746	0.43001	0.47960	0.51332
10	0.36866	0.40925	0.45662	0.48893
11	0.35242	0.39122	0.43670	0.46770
12	0.33815	0.37543	0.41918	0.44905
13	0.32549	0.36143	0.40362	0.43247
14	0.31417	0.34890	0.38970	0.41762
15	0.30397	0.33760	0.37713	0.40420
16	0.29472	0.32733	0.36571	0.39201
17	0.28627	0.31796	0.35528	0.38086
18	0.27851	0.30936	0.34569	0.37062
19	0.27136	0.30143	0.33685	0.36117
20	0.26473	0.29408	0.32866	0.35241
21	0.25858	0.28724	0.32104	0.34427
22	0.25283	0.28087	0.31394	0.33666
23	0.24746	0.27490	0.30728	0.32954
24	0.24242	0.26931	0.30104	0.32286

Practical Reliability Engineering, Fifth Edition. Patrick D. T. O'Connor and Andre Kleyner.

Critical values, $d_{alpha};(n)^a$, of the maximum absolute difference between sample $F_n(x)$ and population $F(x)$ cumulative distribution.

Number of trials, n	Level of significance, α			
	0.10	0.05	0.02	0.01
25	0.23768	0.26404	0.29516	0.31657
26	0.23320	0.25907	0.28962	0.31064
27	0.22898	0.25438	0.28438	0.30502
28	0.22497	0.24993	0.27942	0.29971
29	0.22117	0.24571	0.27471	0.29466
30	0.21756	0.24170	0.27023	0.28987
31	0.21412	0.23788	0.26596	0.28530
32	0.21085	0.23424	0.26189	0.28094
33	0.20771	0.23076	0.25801	0.27677
34	0.20472	0.22743	0.25429	0.27279
35	0.20185	0.22425	0.26073	0.26897
36	0.19910	0.22119	0.24732	0.26532
37	0.19646	0.21826	0.24404	0.26180
38	0.19392	0.21544	0.24089	0.25843
39	0.19148	0.21273	0.23786	0.25518
40^b	0.18913	0.21012	0.23494	0.25205

[a] Values of $d_\alpha(n)$ such that $p(\max)|F^n(x) - F(x)|d^\alpha(n) = \alpha$.

[b] $N > 40 \approx \frac{1.22}{N^{1/2}}, \frac{1.36}{N^{1/2}}, \frac{1.51}{N^{1/2}}$ and $\frac{1.63}{N^{1/2}}$ for the four levels of significance.

TABLE A19 Quantiles of the Smirnov Test Statistic for Two Samples of Equal Size n[a]

One-Sided Test: $p = 0.90$		0.95	0.975	0.99	0.995	One-Sided Test: $p = 0.90$		0.95	0.975	0.99	0.995
Two-Sided Test: $p = 0.80$		**0.90**	**0.95**	**0.98**	**0.99**	**Two-Sided Test: $p = 0.80$**		**0.90**	**0.95**	**0.98**	**0.99**
n = **3**	2/3	2/3				n = **22**	7/22	8/22	8/22	10/22	10/22
4	3/4	3/4	3/4			**23**	7/23	8/23	9/23	10/23	10/23
5	3/5	3/5	4/5	4/5	4/5	**24**	7/24	8/24	9/24	10/24	11/24
6	3/6	4/6	4/6	5/6	5/6	**25**	7/25	8/25	9/25	10/25	11/25
7	4/7	4/7	5/7	5/7	5/7	**26**	7/26	8/26	9/26	10/26	11/26
8	4/8	4/8	5/8	5/8	6/8	**27**	7/27	8/27	9/27	11/27	11/27
9	4/9	5/9	5/9	6/9	6/9	**28**	8/28	9/28	10/28	11/28	12/28
10	4/10	5/10	6/10	6/10	7/10	**29**	8/29	9/29	10/29	11/29	12/29
11	5/11	5/11	6/11	7/11	7/11	**30**	8/30	9/30	10/30	11/30	12/30
12	5/12	5/12	6/12	7/12	7/12	**31**	8/31	9/31	10/31	11/31	12/31
13	5/13	6/13	6/13	7/13	8/13	**32**	8/32	9/32	10/32	12/32	12/32
14	5/14	6/14	7/14	7/14	8/14	**33**	8/33	9/33	11/33	12/33	13/33
15	5/15	6/15	7/15	8/15	8/15	**34**	8/34	10/34	11/34	12/34	13/34
16	6/16	6/16	7/16	8/16	9/16	**35**	8/35	10/35	11/35	12/35	13/35
17	6/17	7/17	7/17	8/17	9/17	**36**	9/36	10/36	11/36	12/36	13/36
18	6/18	7/18	8/18	9/18	9/18	**37**	9/37	10/37	11/37	13/37	13/37
19	6/19	7/19	8/19	9/19	9/19	**38**	9/38	10/38	11/38	13/38	14/38
20	6/20	7/20	8/20	9/20	10/20	**39**	9/39	10/39	11/39	13/39	14/39
21	6/21	7/21	8/21	9/21	10/21	**40**	9/40	10/40	12/40	13/40	14/40
						Approximation for $n > 40$:	$\frac{1.52}{\sqrt{n}}$	$\frac{1.73}{\sqrt{n}}$	$\frac{1.92}{\sqrt{n}}$	$\frac{2.15}{\sqrt{n}}$	$\frac{2.30}{\sqrt{n}}$

SOURCE. Adapted from Birnbaum and Hall (1960), with permission from the Institute of Mathematical Statistics.

[a] The entries in this table are selected quantiles w_p of the Smirnov two-sample test statistic T defined by Equations 6.3.2 and 6.3.3 for the one-tailed test and defined by Equation 6.3.1 for the two-tailed test. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile of T as given in this table. The test statistic is a discrete random variable, so the exact level of significance may be less than the apparent α used in this table.

TABLE A20 Quantiles of the Smirnov Test Statistic for Two Samples of Different Size n and m[a]

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99
$N_1 = 1$	$N_2 = 9$	17/18				
	10	9/10				
$N_1 = 2$	$N_2 = 3$	5/6				
	4	3/4				
	5	4/5	4/5			
	6	5/6	5/6			
	7	5/7	6/7			
	8	3/4	7/8	7/8		
	9	7/9	8/9	8/9		
	10	7/10	4/5	9/10		
$N_1 = 3$	$N_2 = 4$	3/4	3/4			
	5	2/3	4/5	4/5		
	6	2/3	2/3	5/6		
	7	2/3	5/7	6/7	6/7	
	8	5/8	3/4	3/4	7/8	
	9	2/3	2/3	7/9	8/9	8/9
	10	3/5	7/10	4/5	9/10	9/10
	12	7/12	2/3	3/4	5/6	11/12
$N_1 = 4$	$N_2 = 5$	3/5	3/4	4/5	4/5	
	6	7/12	2/3	3/4	5/6	5/6
	7	17/28	5/7	3/4	6/7	6/7
	8	5/8	5/8	3/4	7/8	7/8
	9	5/9	2/3	3/4	7/9	8/9
	10	11/20	13/20	7/10	4/5	4/5
	12	7/12	2/3	2/3	3/4	5/6
	16	9/16	5/8	11/16	3/4	13/16
$N_1 = 5$	$N_2 = 6$	3/5	2/3	2/3	5/6	5/6
	7	4/7	23/35	5/7	29/35	6/7
	8	11/20	5/8	27/40	4/5	4/5
	9	5/9	3/5	31/45	7/9	4/5
	10	1/2	3/5	7/10	7/10	4/5
	15	8/15	3/5	2/3	11/15	11/15
	20	1/2	11/20	3/5	7/10	3/4
$N_1 = 6$	$N_2 = 7$	23/42	4/7	29/42	5/7	5/6
	8	1/2	7/12	2/3	3/4	3/4
	9	1/2	5/9	2/3	13/18	7/9
	10	1/2	17/30	19/30	7/10	11/15
	12	1/2	7/12	7/12	2/3	3/4
	18	4/9	5/9	11/18	2/3	13/18
	24	11/24	1/2	7/12	5/8	2/3

TABLE A20 (Continued)

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99
$N_1 = 7$	$N_2 = 8$	27/56	33/56	5/8	41/56	3/4
	9	31/63	5/9	40/63	5/7	47/63
	10	33/70	39/70	43/70	7/10	5/7
	14	3/7	1/2	4/7	9/14	5/7
	28	3/7	13/28	15/28	17/28	9/14
$N_1 = 8$	$N_2 = 9$	4/9	13/24	5/8	2/3	3/4
	10	19/40	21/40	23/40	27/40	7/10
	12	11/24	1/2	7/12	5/8	2/3
	16	7/16	1/2	9/16	5/8	5/8
	32	13/32	7/16	1/2	9/16	19/32
$N_1 = 9$	$N_2 = 10$	7/15	1/2	26/45	2/3	31/45
	12	4/9	1/2	5/9	11/18	2/3
	15	19/45	22/45	8/15	3/5	29/45
	18	7/18	4/9	1/2	5/9	11/18
	36	13/36	5/12	17/36	19/36	5/9
$N_1 = 10$	$N_2 = 15$	2/5	7/15	1/2	17/30	19/30
	20	2/5	9/20	1/2	11/20	3/5
	40	7/20	2/5	9/20	1/2	—
$N_1 = 12$	$N_2 = 15$	23/60	9/20	1/2	11/20	7/12
	16	3/8	7/16	23/48	13/24	7/12
	18	13/36	5/12	17/36	19/36	5/9
	20	11/30	5/12	7/15	31/60	17/30
$N_1 = 15$	$N_2 = 20$	7/20	2/5	13/30	29/60	31/60
$N_1 = 16$	$N_2 = 20$	27/80	31/80	17/40	19/40	41/80
Large sample approximation		$1.07\sqrt{\frac{m+n}{mn}}$	$1.22\sqrt{\frac{m+n}{mn}}$	$1.36\sqrt{\frac{m+n}{mn}}$	$1.52\sqrt{\frac{m+n}{mn}}$	$1.63\sqrt{\frac{m+n}{mn}}$

SOURCE. Adapted from Massey (1952), with permission from the Institute of Mathematical Statistics.

[a] The entries in this table are selected quantiles w_p of the Smirnov test statistic T for two samples, defined by Equations 6.3.1, 6.3.2, and 6.3.3. To enter the table let N_1 be the smaller sample size and let N_2 be the larger sample size. Reject H_0 at the level α if T exceeds $w_{1-\alpha}$ as given in this table. If n and m are not covered by this table, use the large sample approximation given at the end of the table, or consult exact tables by Kim and Jennrich, which appear in Harter and Owen (1970) for $n, m \leq 100$.

Table XI Critical Values for the Spearman Rho Rank Correlation Coefficient Test

	One-tailed α .05	.025	.01	.005
	Two-tailed α			
n	.10	.05	.02	.01
5	±.900	—	—	—
6	±.829	±.886	±.943	—
7	±.714	±.786	±.893	±.929
8	±.643	±.738	±.833	±.881
9	±.600	±.700	±.783	±.833
10	±.564	±.648	±.745	±.794
11	±.536	±.618	±.709	±.755
12	±.503	±.587	±.678	±.727
13	±.475	±.566	±.672	±.744
14	±.456	±.544	±.645	±.714
15	±.440	±.524	±.622	±.688
16	±.425	±.506	±.601	±.665
17	±.411	±.490	±.582	±.644
18	±.399	±.475	±.564	±.625
19	±.388	±.462	±.548	±.607
20	±.377	±.450	±.534	±.591
21	±.368	±.438	±.520	±.576
22	±.359	±.428	±.508	±.562
23	±.351	±.418	±.496	±.549
24	±.343	±.409	±.485	±.537
25	±.336	±.400	±.475	±.526
26	±.329	±.392	±.465	±.515
27	±.323	±.384	±.456	±.505
28	±.317	±.377	±.448	±.496
29	±.311	±.370	±.440	±.487
30	±.305	±.364	±.432	±.478

Upper Critical Values for Kendall's Rank Correlation Coefficient $\hat{\tau}$

Note: In the table below, the critical values give significance levels as close as possible to but not exceeding the nominal α.

	Nominal α					
n	0.10	0.05	0.025	0.01	0.005	0.001
4	1.000	1.000	-	-	-	-
5	0.800	0.800	1.000	1.000	-	-
6	0.600	0.733	0.867	0.867	1.000	-
7	0.524	0.619	0.714	0.810	0.905	1.000
8	0.429	0.571	0.643	0.714	0.786	0.857
9	0.389	0.500	0.556	0.667	0.722	0.833
10	0.378	0.467	0.511	0.600	0.644	0.778
11	0.345	0.418	0.491	0.564	0.600	0.709
12	0.303	0.394	0.455	0.545	0.576	0.667
13	0.308	0.359	0.436	0.513	0.564	0.641
14	0.275	0.363	0.407	0.473	0.516	0.604
15	0.276	0.333	0.390	0.467	0.505	0.581
16	0.250	0.317	0.383	0.433	0.483	0.567
17	0.250	0.309	0.368	0.426	0.471	0.544
18	0.242	0.294	0.346	0.412	0.451	0.529
19	0.228	0.287	0.333	0.392	0.439	0.509
20	0.221	0.274	0.326	0.379	0.421	0.495
21	0.210	0.267	0.314	0.371	0.410	0.486
22	0.203	0.264	0.307	0.359	0.394	0.472
23	0.202	0.257	0.296	0.352	0.391	0.455
24	0.196	0.246	0.290	0.341	0.377	0.449
25	0.193	0.240	0.287	0.333	0.367	0.440
26	0.188	0.237	0.280	0.329	0.360	0.428
27	0.179	0.231	0.271	0.322	0.356	0.419
28	0.180	0.228	0.265	0.312	0.344	0.413
29	0.172	0.222	0.261	0.310	0.340	0.404

	Nominal α					
n	0.10	0.05	0.025	0.01	0.005	0.001
30	0.172	0.218	0.255	0.301	0.333	0.393
31	0.166	0.213	0.252	0.295	0.325	0.389
32	0.165	0.210	0.246	0.290	0.323	0.379
33	0.163	0.205	0.242	0.288	0.314	0.375
34	0.159	0.201	0.237	0.280	0.312	0.369
35	0.156	0.197	0.234	0.277	0.304	0.361
36	0.152	0.194	0.232	0.273	0.302	0.359
37	0.150	0.192	0.228	0.267	0.297	0.351
38	0.149	0.189	0.223	0.263	0.292	0.346
39	0.147	0.188	0.220	0.260	0.287	0.341
40	0.144	0.185	0.218	0.256	0.285	0.338
41	0.141	0.180	0.215	0.254	0.280	0.334
42	0.141	0.178	0.213	0.250	0.275	0.329
43	0.138	0.176	0.209	0.247	0.274	0.324
44	0.137	0.173	0.207	0.243	0.268	0.321
45	0.135	0.172	0.204	0.240	0.267	0.317
46	0.132	0.169	0.202	0.239	0.264	0.314
47	0.132	0.167	0.199	0.236	0.260	0.310
48	0.129	0.167	0.197	0.232	0.257	0.307
49	0.129	0.163	0.196	0.230	0.253	0.303
50	0.127	0.162	0.192	0.228	0.251	0.300
51	0.126	0.161	0.191	0.225	0.249	0.297
52	0.124	0.158	0.189	0.223	0.246	0.294
53	0.123	0.157	0.187	0.221	0.244	0.290
54	0.122	0.156	0.185	0.219	0.241	0.287
55	0.121	0.154	0.182	0.216	0.239	0.285
56	0.119	0.152	0.181	0.214	0.236	0.282
57	0.118	0.152	0.179	0.212	0.234	0.279
58	0.117	0.149	0.177	0.210	0.232	0.276
59	0.116	0.148	0.176	0.209	0.230	0.274
60	0.115	0.147	0.174	0.207	0.228	0.272

- ooo00ooo -